T-odd Asymmetry in W + jet events at the LHC

National Central University

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in collaboration with

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Phys. Rev. Lett. 113, 152001 (2014) [arXiv:1407.1016 [hep-ph]]

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Introduction

Absorptive Part of Scattering Amplitude

We study **theoretical calculation** & **direct measurement** of the absorptive part of a QCD amplitude.

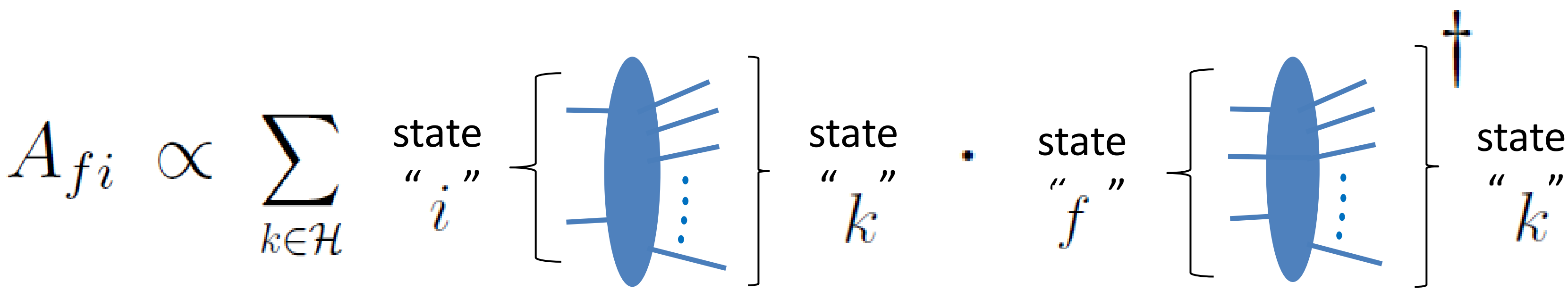
Transition operator $\hat{T}$ is given as $\hat{S} = T\left[e^{-i\int \mathrm{d}^4x \hat{\mathcal{H}}_{\mathrm{int}}}\right] = \hat{1} + i\hat{T}$.

Unitarity of S-matrix $\hat{S}^\dagger\hat{S} = \hat{1}$ gives $-i(\hat{T} - \hat{T}^\dagger) = \hat{T}^\dagger\hat{T}$.

Absorptive part of transition amplitude from state "i" to state "f" is defined as

$$-i\langle f|(\hat{T} - \hat{T}^\dagger)|i\rangle = \langle f|\hat{T}^\dagger\hat{T}|i\rangle = \sum_{k\in\mathcal{H}} \langle f|\hat{T}^\dagger|k\rangle\langle k|\hat{T}|i\rangle \equiv A_{fi}$$

summation over complete set of states $\mathcal{H}$ →



c.f. When the initial and final states are the same, we get the optical theorem:

$$A_{ii} = \mathrm{Im}\left(\langle i|\hat{T}|i\rangle\right) = \sum_{k\in\mathcal{H}} |\langle k|\hat{T}|i\rangle|^2 \propto \sigma_{\mathrm{tot}}$$

Measurement of Absorptive Part

Absorptive part can be measured through part of the cross section that is odd under naïve T-reversal, which we call "T-odd asymmetry".

any 3-momentum $\vec{k} \rightarrow -\vec{k}$, any spin $\sigma \rightarrow -\sigma$, but unlike true T-reversal, the initial and final states are not interchanged:

$$\langle \vec{p'}_j,\ \vec{\sigma'}_j | \hat{S} | \vec{p}_i,\ \vec{\sigma}_i \rangle \longrightarrow \langle -\vec{p'}_j,\ -\vec{\sigma'}_j | \hat{S} | -\vec{p}_i,\ -\vec{\sigma}_i \rangle$$

free particle states

proof:

We denote the naïve-T-reversal of states i , f by $\tilde{i}$, $\tilde{f}$, respectively.

We find $|\langle \tilde{f} | \hat{T} | \tilde{i} \rangle|^2 - |\langle f | \hat{T} | i \rangle|^2$ ← T-odd asymmetry

$$= \underbrace{\left(|\langle \tilde{f} | \hat{T} | \tilde{i} \rangle|^2 - |\langle i | \hat{T} | f \rangle|^2 \right)}_{\text{Odd under original T-reversal}} + 2\,\mathrm{Im}\left(\langle f | \hat{T} | i \rangle^* A_{fi} \right) + |A_{fi}|^2$$

Absorptive part

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$$= \cancel{\left(|\langle \tilde{f} | \hat{T} | \tilde{i} \rangle|^2 - |\langle i | \hat{T} | f \rangle|^2 \right)} + 2\,\mathrm{Im}\left(\langle f | \hat{T} | i \rangle^* A_{fi} \right) + |A_{fi}|^2$$

Absorptive part

In QCD with $\Theta = 0$, the first term is zero, but the second and third terms are significant as hadrons/partons have much chance of rescattering, due to the strongly-coupled nature of QCD.

Observation of T-odd Asymmetry

T-odd asymmetry can be observed through distribution of a quantity that is odd under naïve-T-reversal, such as

$$\vec{k}_1 \times \vec{k}_2 \cdot \vec{k}_3 \quad , \qquad \vec{k}_1 \times \vec{k}_2 \cdot \vec{s} \quad .$$

$\vec{k}_i$: 3-momentum
$\vec{s}$: spin vector

$\vec{k}_1 \times \vec{k}_2 \cdot \vec{k}_3$ is P-odd and $\vec{k}_1 \times \vec{k}_2 \cdot \vec{s}$ is P-even. Absorptive part of QCD amplitudes arises as asymmetry of $\vec{k}_1 \times \vec{k}_2 \cdot \vec{s}$.

Early proposals for its observation are

De Rujula, Kaplan & de Rafael (1971)

$e^- p_{\text{trans. polarized}} \to e^- p$: measure up and down-ward asymmetry of scattered e^-.

De Rujula, Petronzio & Lautrup (1978)

$e^-_{\text{polarized}} e^+ \to \gamma^* \to (\text{vector resonance}) \to g\,g\,g$: measure $\vec{k}_{j1} \times \vec{k}_{j2} \cdot \vec{s}_{e^-}$.

Hagiwara, Hikasa & Kai (1982)

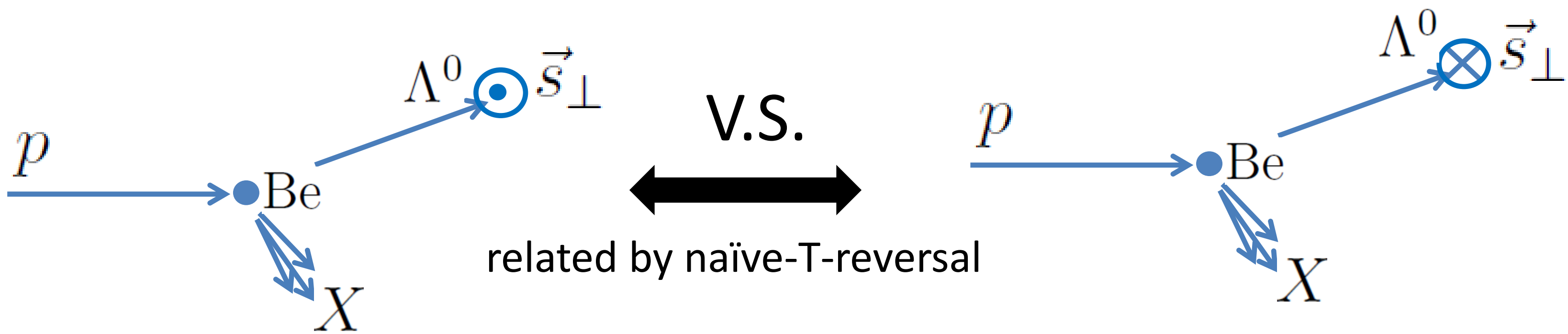
$\nu + N \to \ell + (\text{hadron}) + \text{anything}$: measure $\vec{k}_\ell \times \vec{k}_{\text{hadron}} \cdot \vec{s}_\nu$.

These are followed by works on W+jet production in hadron collisions (Hagiwara, Hikasa & Kai (1984), Hagiwara, Hikasa & Yokoya (2006)), Z-boson decay (Hagiwara, Kuruma & Y. Yamada (1991)) and top decay (Hagiwara, Mawatari & Yokoya (2007)).

Past Observation of T-odd Asymmetry

Bunce *et al.* (1975)

- T-odd asymmetry has already been observed through transverse polarization of Λ^0 in $p\,\mathrm{Be} \to \Lambda^0 X$ process, which is given by $\vec{s}_\perp \propto \vec{p}_p \times \vec{p}_{\Lambda^0} \cdot \vec{s}_{\Lambda^0}$.



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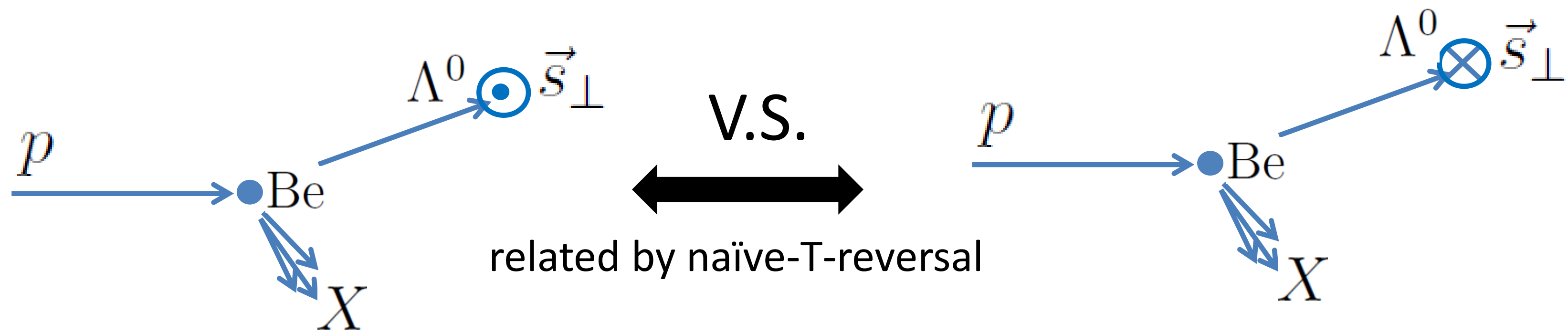


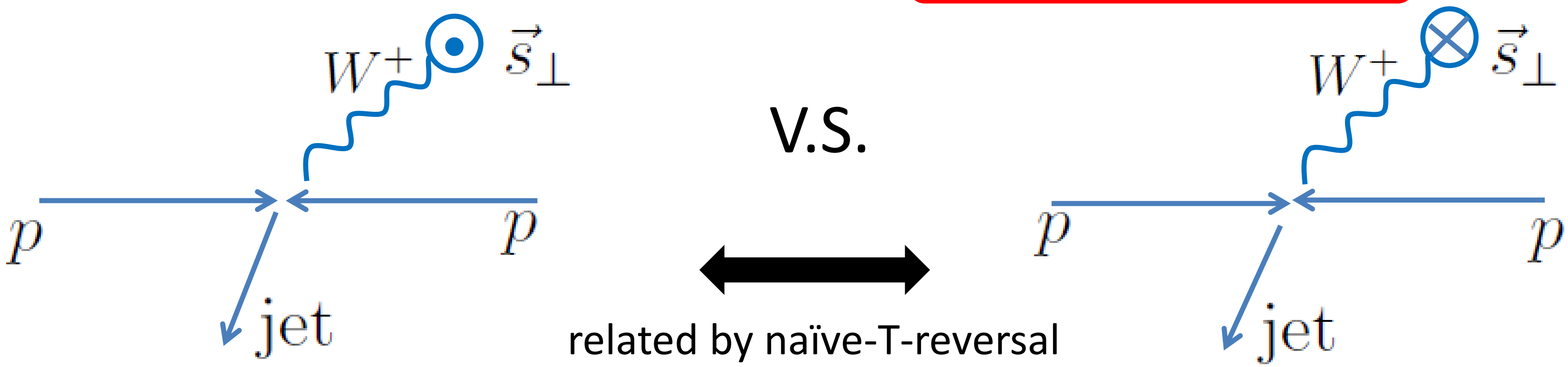
- However, it is difficult to make a theoretical prediction on the transverse polarization of Λ^0 .

In this talk, we focus on transverse polarization of W^+ in $pp \to W^+ + \mathrm{jet}$ process, which can be predicted by perturbative QCD.

T-odd Asymmetry in $pp \to W^+ + \mathrm{jet}$ Events

T-odd Asymmetry in $W + \mathrm{jet}$ events

In this talk, we focus on $\boxed{p\,p \;\rightarrow\; W^+ \;+\; \mathrm{jet}}$ process at the LHC and study asymmetry of the quantity $\boxed{\vec{p}_{p1} \times \vec{p}_{W^+} \cdot \vec{s}_\perp}$.



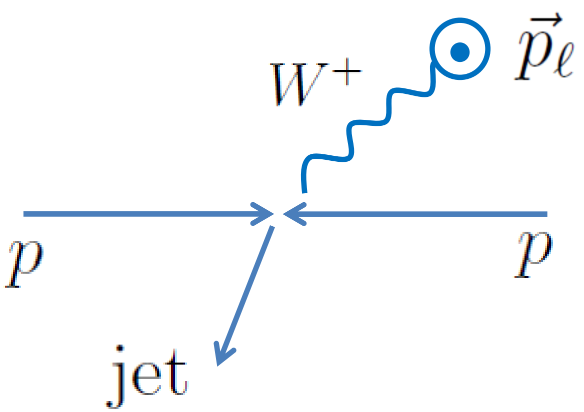
Since W boson is heavy, we may apply **perturbative QCD** to calculate the scattering amplitude, including its absorptive part.

In perturbation theory, absorptive part is calculated by Cutkosky rules.

Observation of T-odd Asymmetry in $W + \text{jet}$ events

The transverse spin of W^+, $\vec{s}_\perp$, can be inferred through its decay into leptons $W^+ \to \ell^+ \nu_\ell$, thanks to P-violation in weak interaction.

We study the momentum distribution of the charged lepton ℓ^+. $\vec{s}_\perp$ is correlated with the component of ℓ^+ momentum perpendicular to the scattering plane.



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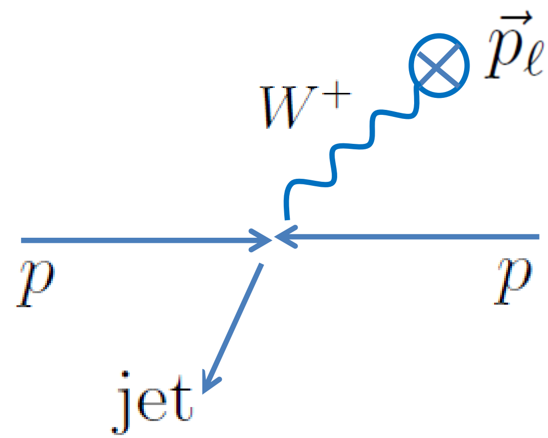


Differential Cross Section of $pp \to W^+(\to \ell^+ \nu_\ell) + \text{jet}$ Process

The differential cross section is expressed as

(diff. cross section) $\propto \sum_{\lambda,\lambda'=\pm,0} \mathcal{M}_{pp\to W_\lambda + \text{jet}} \mathcal{M}_{W_\lambda \to \ell\nu} \mathcal{M}^*_{pp\to W_{\lambda'} + \text{jet}} \mathcal{M}^*_{W_{\lambda'} \to \ell\nu}$

$$= \sum_{\lambda,\lambda'=\pm,0} D_{\lambda\lambda'} \, \rho_{\lambda\lambda'}$$

(λ, λ' = W boson polarization)

where $D_{\lambda\lambda'} \equiv \mathcal{M}_{pp\to W_\lambda + \text{jet}} \mathcal{M}^*_{pp\to W_{\lambda'} + \text{jet}}$, ← QCD amplitude incl. absorptive part

$$\rho_{\lambda\lambda'} \equiv \mathcal{M}_{W_\lambda \to \ell\nu} \mathcal{M}^*_{W_{\lambda'} \to \ell\nu}$$

$$\propto \begin{pmatrix} \frac{(1+\cos\theta)^2}{4} & e^{i\phi}\frac{(1+\cos\theta)\sin\theta}{2\sqrt{2}} & -e^{2i\phi}\frac{\sin^2\theta}{4} \\ e^{-i\phi}\frac{(1+\cos\theta)\sin\theta}{2\sqrt{2}} & \frac{\sin^2\theta}{2} & e^{i\phi}\frac{(1-\cos\theta)\sin\theta}{2\sqrt{2}} \\ -e^{-2i\phi}\frac{\sin^2\theta}{4} & e^{-i\phi}\frac{(1-\cos\theta)\sin\theta}{2\sqrt{2}} & \frac{(1-\cos\theta)^2}{2} \end{pmatrix} .$$

← W boson density matrix

(θ, ϕ) : solid angle of charged lepton in a W boson rest frame

Differential Cross Section

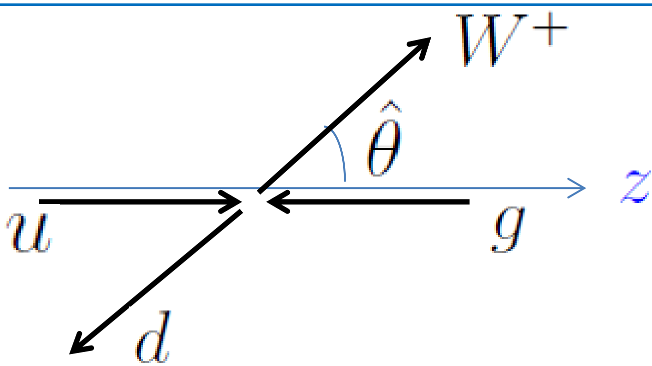
$$\frac{d\sigma}{dq_T^2 d\cos\hat{\theta} d\cos\theta d\phi} = F_1(1+\cos^2\theta) + F_2(1-3\cos^2\theta) + F_3 \sin 2\theta \cos\phi$$
$$+ F_4 \sin^2\theta \cos 2\phi + F_5 \cos\theta + F_6 \sin\theta\cos\phi$$
$$+ F_7 \sin\theta \sin\phi + F_8 \sin 2\theta \sin\phi + F_9 \sin^2\theta \sin 2\phi \ .$$

q_T : W boson transverse momentum

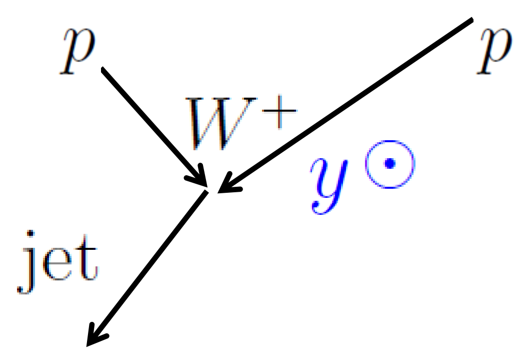
$\hat{\theta}$: scattering angle of W boson in parton center-of-mass frame

(θ, ϕ) : solid angle of charged lepton in a W rest frame with $\vec{n}_y \, // \, \vec{n}_z^{\text{lab}} \times \vec{q}_T^{\text{lab}}$

parton center-of-mass frame



a W rest frame



F_i's are functions of q_T, $\cos\hat{\theta}$, and contain QCD amplitudes and parton distribution functions.

T-odd Part of the Cross Section

$$\frac{\mathrm{d}\sigma}{\mathrm{d}q_T^2 \mathrm{d}\cos\hat{\theta} \mathrm{d}\cos\theta \mathrm{d}\phi} = F_1(1+\cos^2\theta) + F_2(1-3\cos^2\theta) + F_3 \sin 2\theta \cos\phi$$
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Under naïve-T-reversal, $\hat{\theta} \rightarrow \hat{\theta}$, $\theta \rightarrow \theta$, $\phi \rightarrow -\phi$.

Hence functions $F_7, \, F_8, \, F_9$ represent T-odd contributions to the cross section.

T-odd Part of the Cross Section

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$$+ F_7 \sin\theta \sin\phi + F_8 \sin 2\theta \sin\phi + F_9 \sin^2\theta \sin 2\phi \,.$$

Odd under naïve-T-reversal

q_T : W boson transverse momentum

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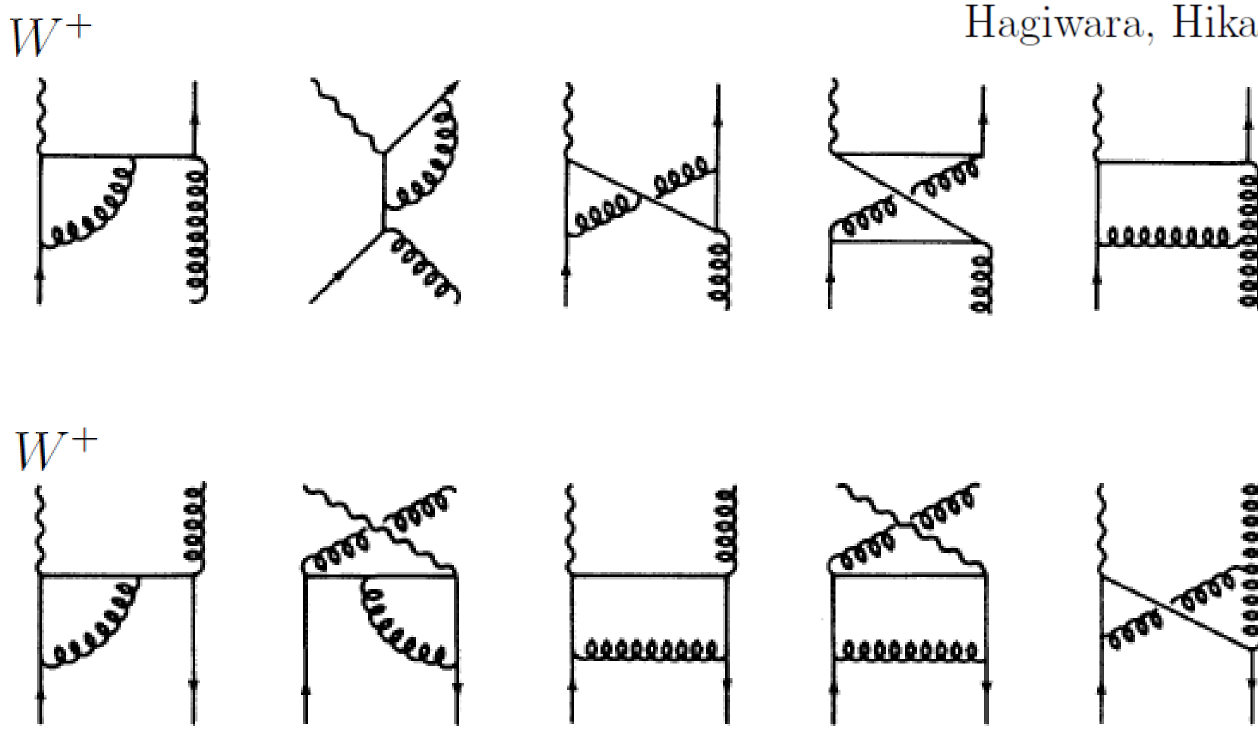
Hence functions F_7, F_8, F_9 represent T-odd contributions to the cross section.

Calculation of T-odd Terms

- At the leading order, T-odd terms $F_7,\ F_8,\ F_9$ can be calculated by inserting Cutkosky cuts into one-loop diagrams below:

Hagiwara, Hikasa & Kai (1984)



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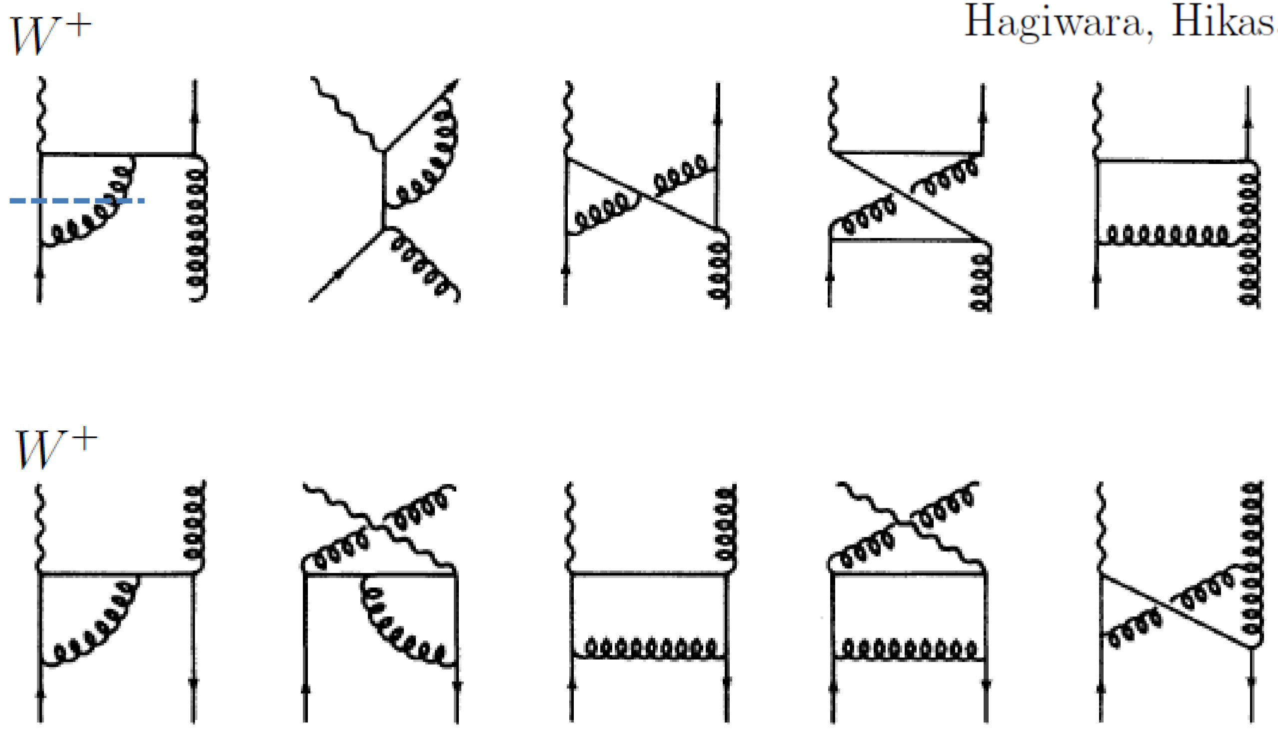

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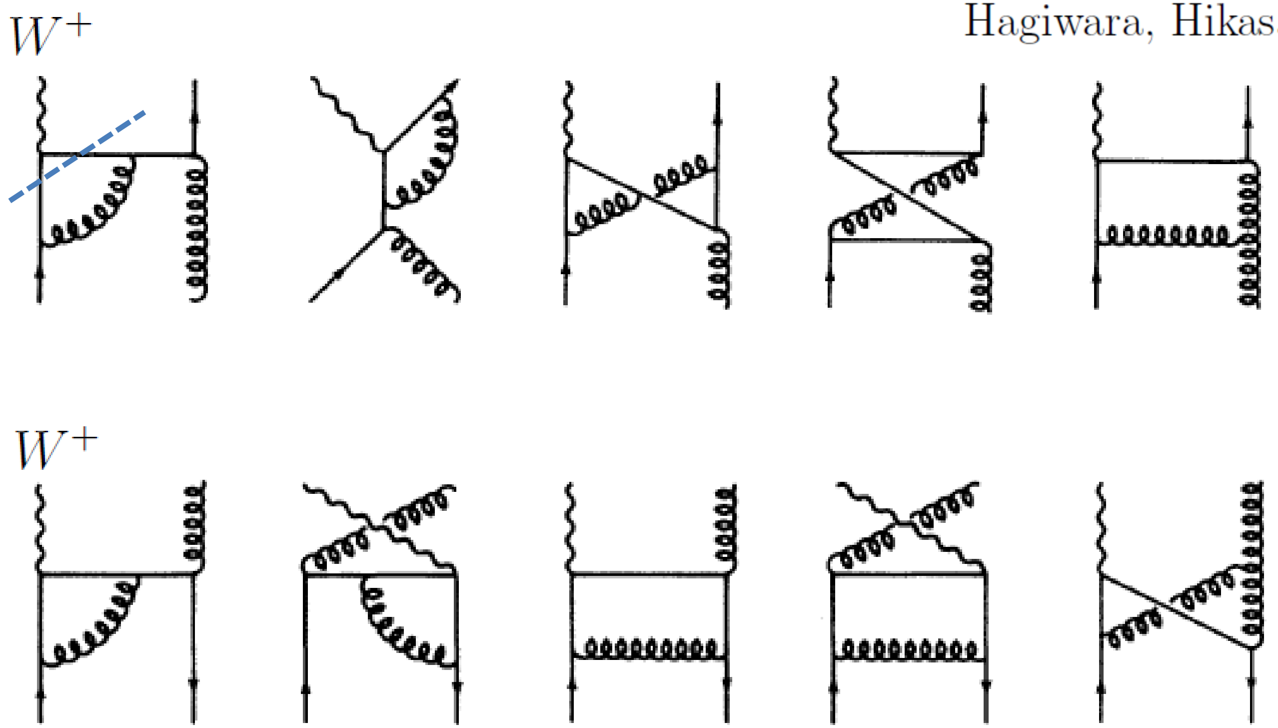

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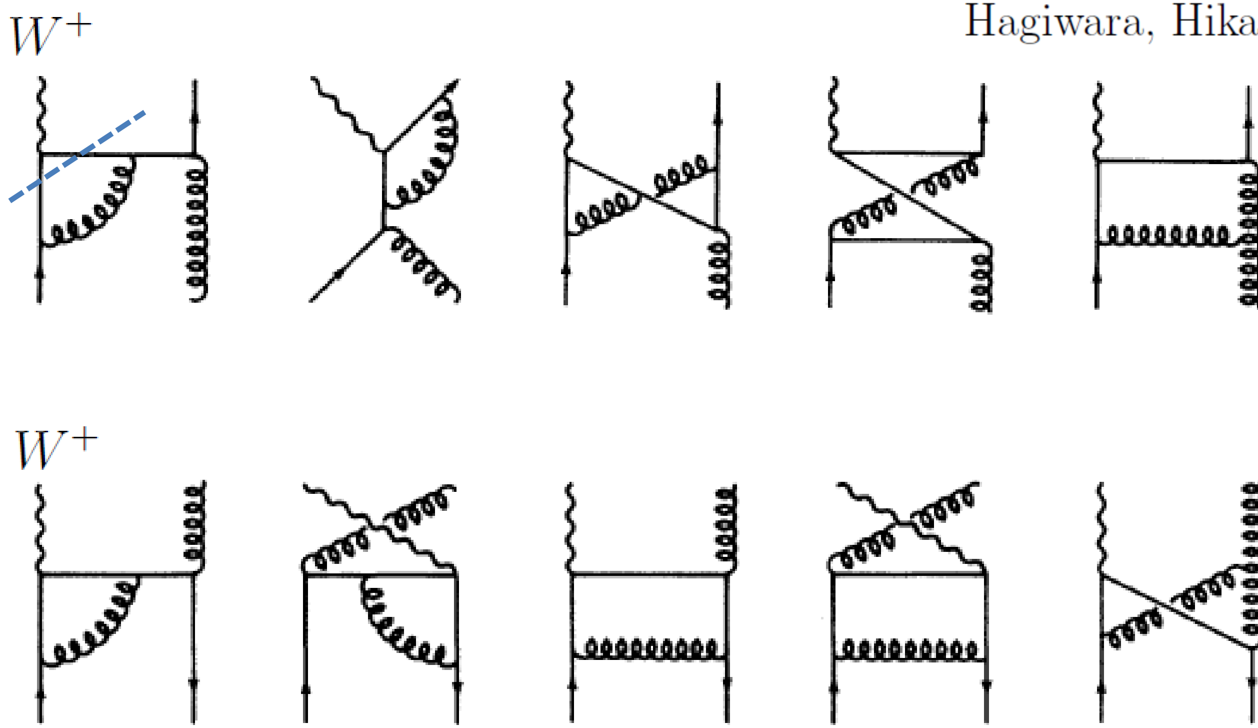
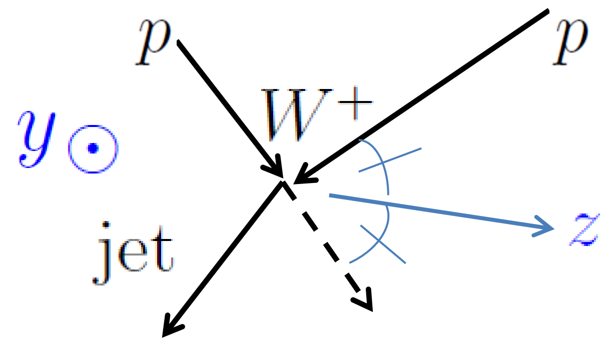


For each cut propagator, replace $\dfrac{1}{(l+p)^2 - m^2 + i\epsilon}$ with $-2\pi i\,\delta((l+p)^2 - m^2)$.

Calculation of T-odd Terms (cont'd)

- Before the calculation, we fix the W boson rest frame where (θ, ϕ) are defined to be Collins-Soper frame:



- Factorize F_i ($i = 7, 8, 9$) as

c = parton

$$F_i = \sum_{\text{partons}\, a,b} \int \mathrm{d}Y\, D_{a/p}(x_+, \mu_F) D_{b/p}(x_-, \mu_F) \frac{3\mathrm{Br}(W \to \ell\nu) G_F M_W^2}{4\sqrt{2} s (\hat{s} + M_W^2) \sin^2\hat{\theta}}\, f_i^{ab \to W^+ c}\left(\frac{M_W^2}{2q \cdot p_a}, \frac{M_W^2}{2q \cdot p_b}\right)$$

where $x_\pm = \dfrac{q_T + \sqrt{q_T^2 + M_W^2 \sin^2\hat{\theta}}}{\sqrt{s}\sin\hat{\theta}} e^{\pm Y}$.

q : W boson momentum
q_T : W boson transverse momentum

Calculation of T-odd Terms (cont'd)

For $ug \to W^+ d$, $\bar{d}g \to W^+ \bar{u}$ subprocesses,

$$f_i^{\bar{d}g\to W^+\bar{u}}(a,\,b) \;=\; \eta_i\, f_i^{ug\to W^+ d}(a,\,b) = -\cos^2\theta_C \frac{\alpha_s(\mu_R)^2}{\pi}\frac{T_F}{N_C} f_{Ci}(a,\,b)\,,$$

where

$$f_{C7} = \frac{1-b}{[c(1-c)]^{1/2}}\left\{-C_F\left[\frac{a(1+a)c}{2b} - a^2\right] + C_1\left[2bc - a^2 + (c-b)\left(1 + \frac{a}{c}\ln\frac{1}{1-c} + \frac{a}{c-a}\ln\frac{a}{c}\right)\right]\right\}\,,$$

$$f_{C8} = \frac{c-b}{2(1-c)^{1/2}}\left\{-C_F\left(\frac{a}{b} - \frac{1+a}{2}\right) + C_1\left[b - 1 + \frac{a}{c}\left(b + \frac{c-b}{c}\ln\frac{1}{1-c}\right)\right]\right\}\,,$$

$$f_{C9} = \frac{c-b}{2c^{1/2}}\left\{-C_F\left(\frac{a}{2b} + \frac{1+a}{2}\right) + C_1\left[b + \frac{a}{c}\ln\frac{1}{1-c} - \frac{1}{1-a}\left(1 + \frac{a}{c-a}\ln\frac{a}{c}\right)\right]\right\}\,.$$

$$c \equiv a + b - ab\,.$$
$$\eta_7 = 1,$$
$$\eta_8 = \eta_9 = -1\,.$$
$$C_1 = C_F - C_A/2\,.$$

For $u\bar{d} \to W^+ g$ subprocess,

$$f_i^{\bar{d}u\to W^+ g}(a,\,b) \;=\; \eta_i\, f_i^{u\bar{d}\to W^+ g}(a,\,b) = -\cos^2\theta_C \frac{\alpha_s(\mu_R)^2}{\pi}\frac{C_F}{N_C} f_{Ai}(a,\,b),$$

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$$f_{A8} = \frac{c}{2(1-c)^{1/2}}\left\{-C_F\frac{a}{b} + C_1\frac{1}{1-a}\ln\frac{a}{c}\right\} - (a \leftrightarrow b)\,,$$

$$f_{A9} = \frac{c^{1/2}}{2}\left\{-C_F\frac{a}{2b} - C_1\frac{1}{1-a}\left[1 - \frac{c}{c-a}\ln\frac{a}{c}\right]\right\} + (a \leftrightarrow b)\,.$$

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T-odd terms appear at α_s^2.

Calculation of T-odd Terms (cont'd)

We plot the ratio $A_i \equiv F_i/F_1 \quad (i = 7, 8, 9)$.

(total cross section is proportional to F_1)

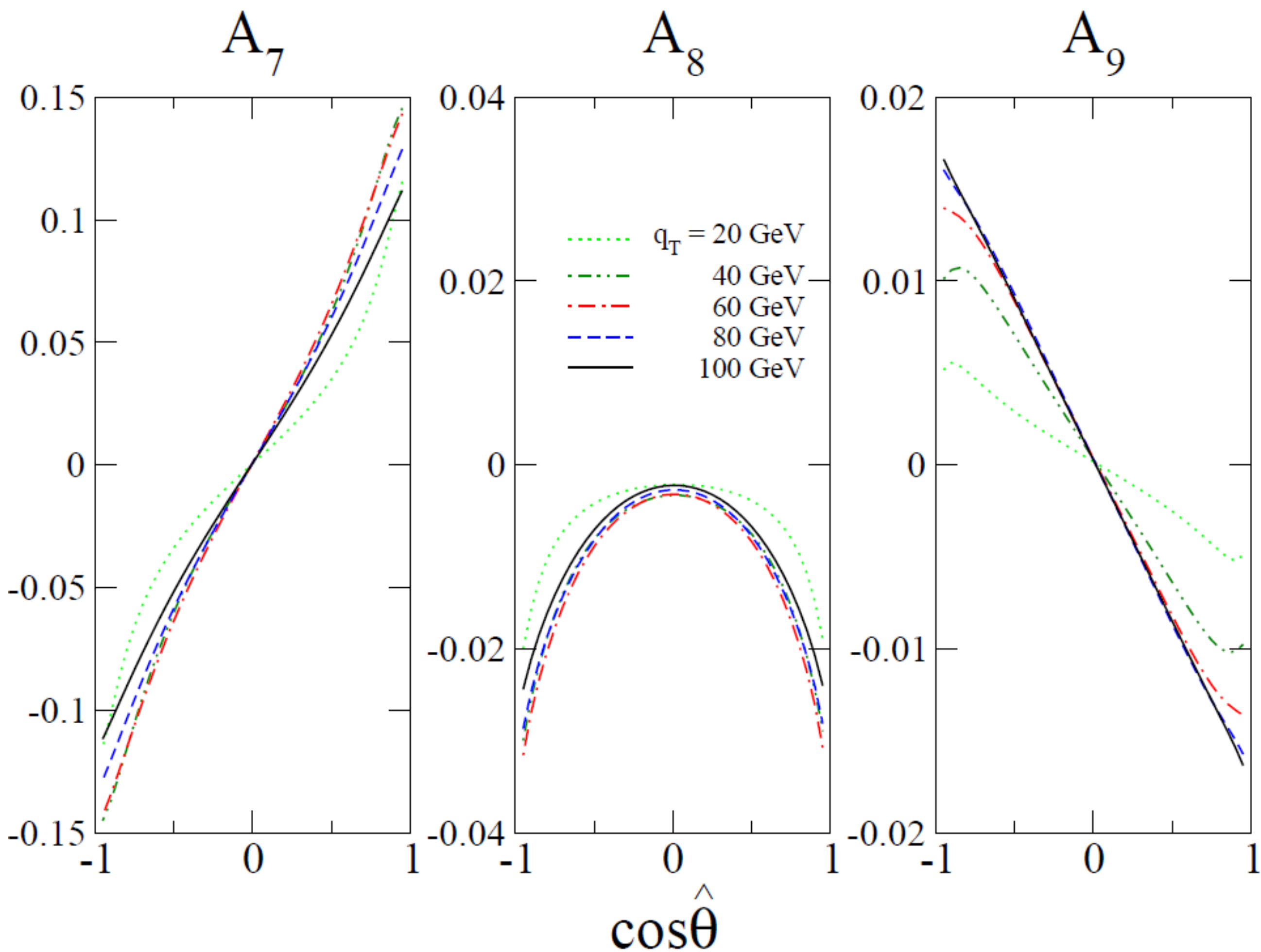
Yokoya (2013)

In pp collisions with $\sqrt{s} = 8$ TeV ,

$$A_i(q_T, \cos\hat{\theta})$$

are evaluated as:

Fac. & ren. scales are set at $\mu_F = \mu_R = q_T$.



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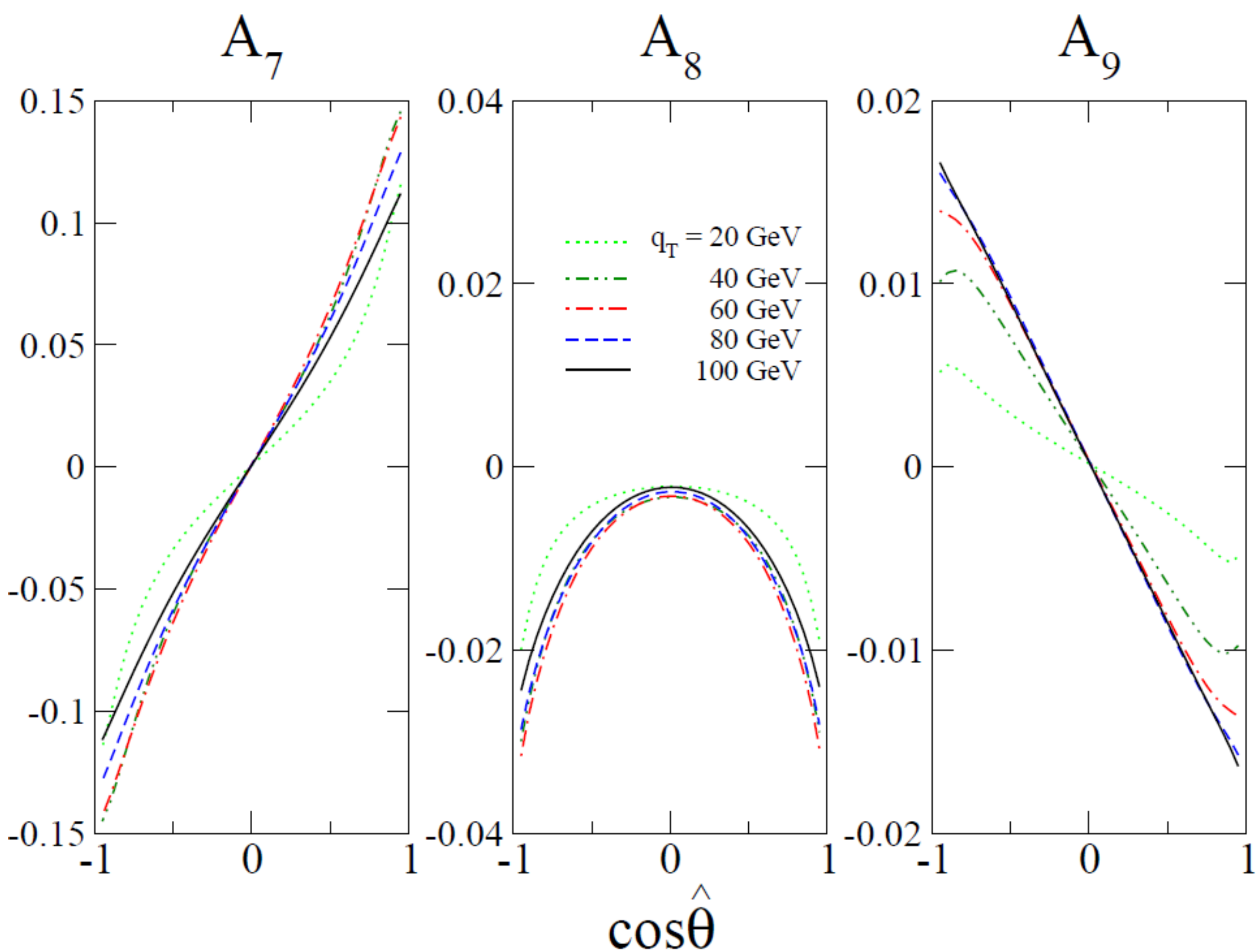

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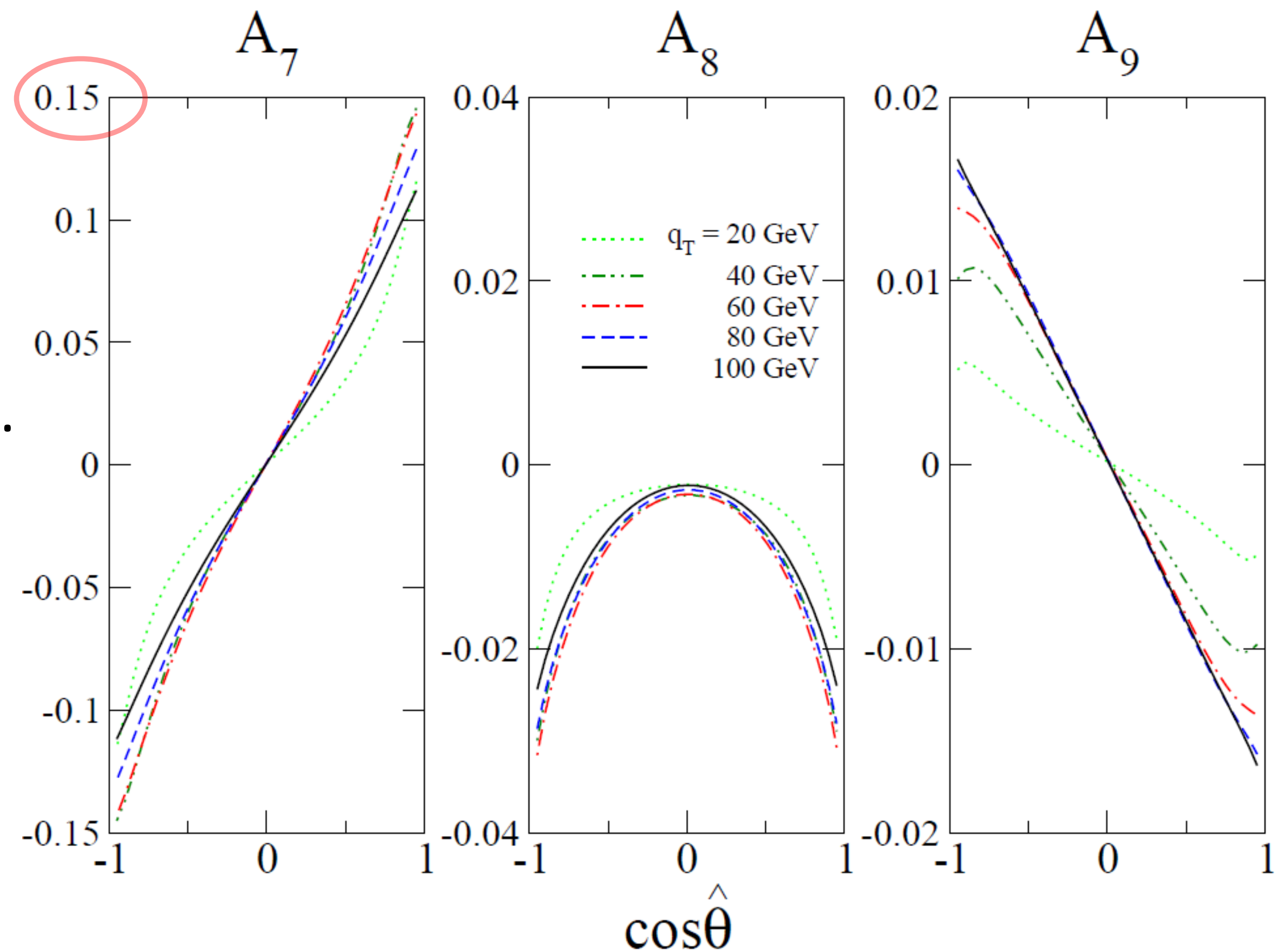
Yokoya (2013)



Calculation of T-odd Terms (cont'd)

We plot the ratio $A_i \equiv F_i/F_1 \quad (i = 7, 8, 9)$.

(total cross section is proportional to F_1)

In pp collisions with $\sqrt{s} = 8\ \mathrm{TeV}$,

$A_i(q_T, \cos\hat{\theta})$

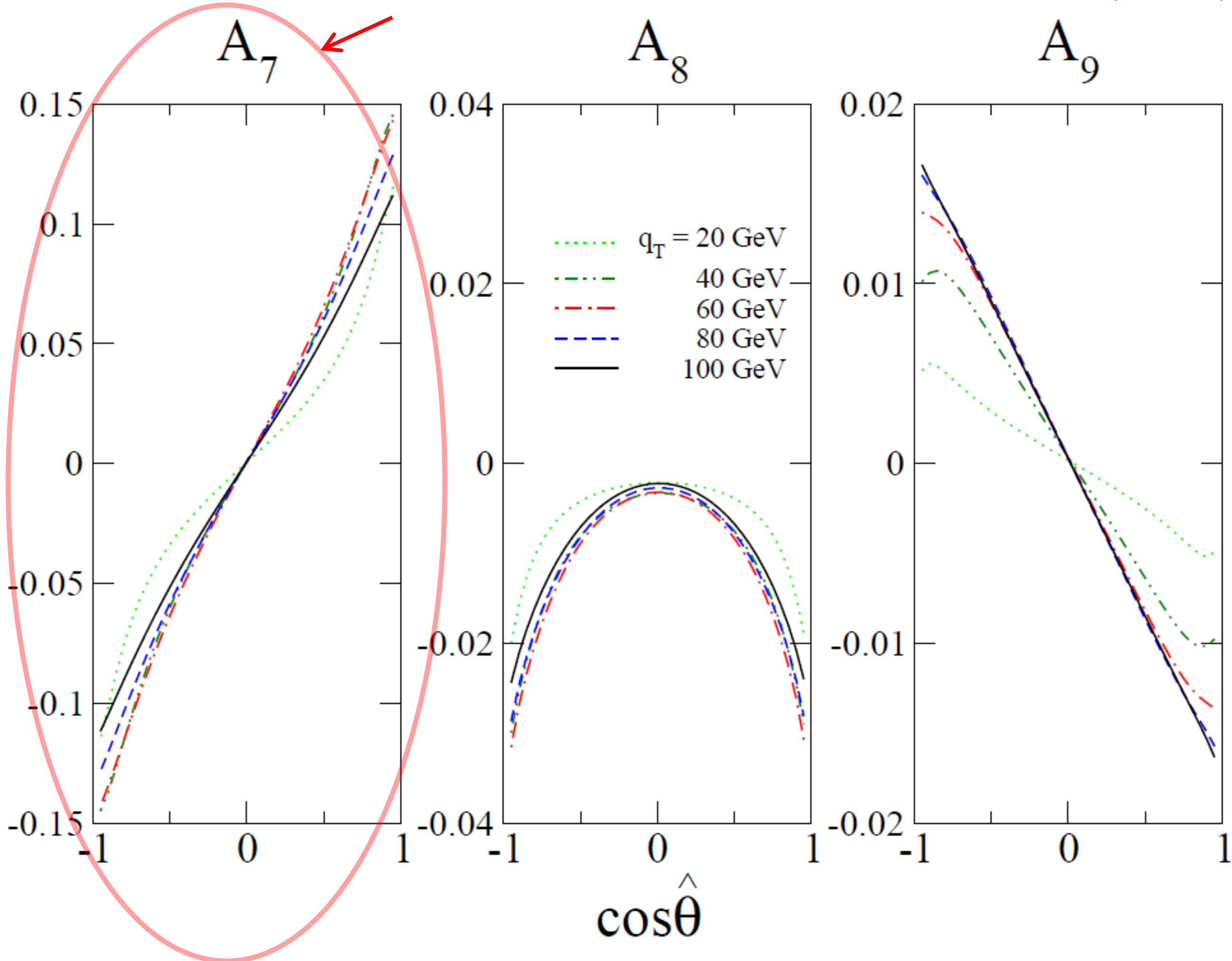
are evaluated as:

Fac. & ren. scales are set at $\mu_F = \mu_R = q_T$.

(A)symmetry of A_i's in terms of $\cos\hat{\theta}$ results from rotational invariance of S-matrix.

Focus on the dominant A_7 term.

Yokoya (2013)



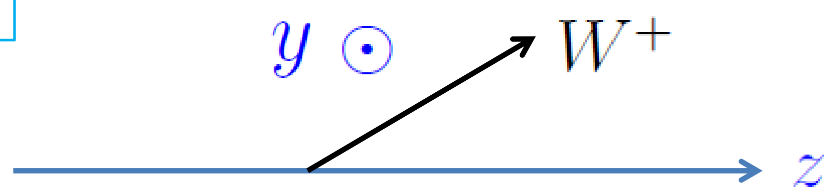
T-odd Observable at Hadron Colliders

We look for a quantity that is sensitive to $F_7 \sin\theta \sin\phi$ term.

When y -axis is defined parallel to $\vec{n}_z^{\,\mathrm{lab}} \times \vec{q}_T^{\,\mathrm{lab}}$, the y-component of the charged lepton momentum, $(\vec{q}_l)_y$, satisfies

$$\boxed{\sin\theta \sin\phi \ = \ (\vec{q}_l)_y \,/\, (M_W/2)} \,.$$

Lab. frame

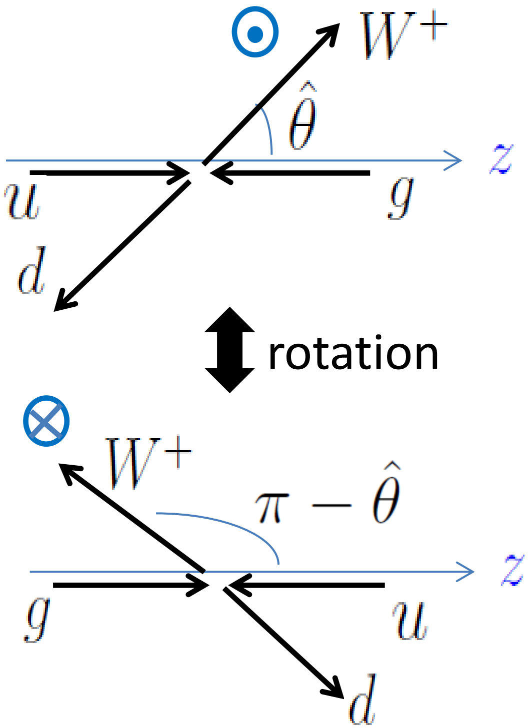


Measure the difference in the numbers of events with $(\vec{q}_l)_y > 0$ and $(\vec{q}_l)_y < 0$, and define **T-odd asymmetry** A as

$$A \ \equiv \ \frac{\sigma(\text{events with } (\vec{q}_l)_y > 0) \ - \ \sigma(\text{events with } (\vec{q}_l)_y < 0)}{\sigma(\text{events with } (\vec{q}_l)_y > 0) \ + \ \sigma(\text{events with } (\vec{q}_l)_y < 0)} \,.$$

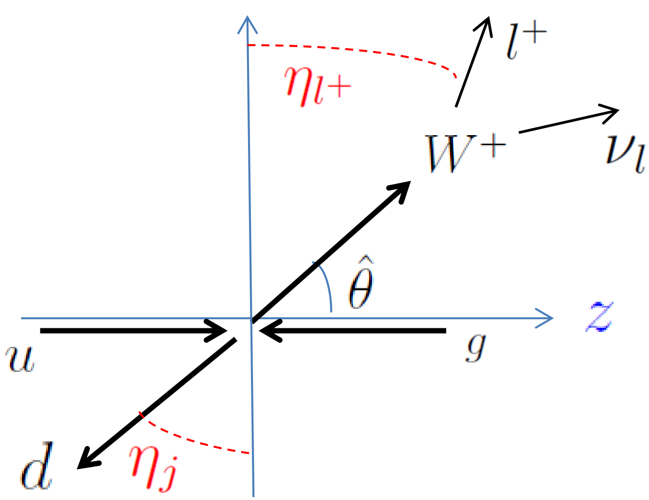
A reflects $F_7 \sin\theta \sin\phi$ term's contribution.

Problem of the Sign of $\cos\hat{\theta}$



F_7 term **flips sign** with cosine of the scattering angle in parton center-of-mass frame $\cos\hat{\theta}$. Hence we need to separate events with $\cos\hat{\theta} > 0$ and $\cos\hat{\theta} < 0$.

In hadron collisions, $\cos\hat{\theta}$ is reconstructed by calculating ν_l's longitudinal momentum, but this gives one positive and one negative solutions for $\cos\hat{\theta}$.



Instead, we use $\eta_{\ell^+} - \eta_j$, which is correlated with $\cos\hat{\theta}$.

We study the distribution of the asymmetry A in each bin of the pseudo-rapidity difference $\eta_{l^+} - \eta_j$.

Simulations for the 8 TeV LHC

Parton-level Analysis

Functions $F_i(q_T, \cos\hat{\theta})$ $(i = 1, 2, ..., 9)$ in the leading order have been calculated by perturbative QCD in Collins-Soper frame in refs. M.Chaichian *et al.* (1982) , Hagiwara, Hikasa & Kai (1984) .
(F_7, F_8, F_9 at one-loop level, the others at tree level)

Integrating the analytic formulas in the references above, we evaluate the differential cross section at **8 TeV LHC**. ⇨ Parton-level analysis

In the following analysis, CTEQ6M parton distribution function is used, and the renormalization and factorization scales are set at $\mu_R = \mu_F = q_T$.

Parton-level Analysis (cont'd)

The selection cuts are

- *The leading jet(=parton) should satisfy* $p_{j_1 T} > 30\ \mathrm{GeV}$ *&* $|\eta_{j_1}| < 4.4$.
- *Require one* μ^+ *with* $p_{\mu^+ T} > 25\ \mathrm{GeV}$ *&* $|\eta_{\mu^+}| < 2.4$.
- *Require* $\not{p}_T > 25\ \mathrm{GeV}$.
- *The transverse momentum of W boson,* $q_T \equiv |\vec{p}_{\mu T} + \vec{\not{p}}_T|$ *, should satisfy* $q_T > 30\ \mathrm{GeV}$.
- *The transverse mass,* $M_T \equiv \sqrt{2(\,|\vec{p}_{\mu T}|\,|\vec{\not{p}}_T| - \vec{p}_{\mu T}\cdot\vec{\not{p}}_T\,)}$ *, should satisfy* $M_T > 60\ \mathrm{GeV}$.

defining $pp \to W^+(\mu^+\nu_\mu) + \mathrm{jet}$ events.

- *Require* $|(\vec{q}_l)_y| \,/\, (M_W/2) > 0.6$.

to reduce the impact of uncertainty of $|(\vec{q}_l)_y|$.

For each event, $(\vec{q}_l)_y$ is reconstructed as:

$$|(\vec{q}_l)_y| = \left|\vec{p}_{\mu T} - \vec{q}_T \frac{\vec{q}_T\cdot\vec{p}_{\mu T}}{|\vec{q}_T|^2}\right| \longleftarrow \vec{q}_T \equiv \vec{p}_{\mu T} + \vec{\not{p}}_T$$

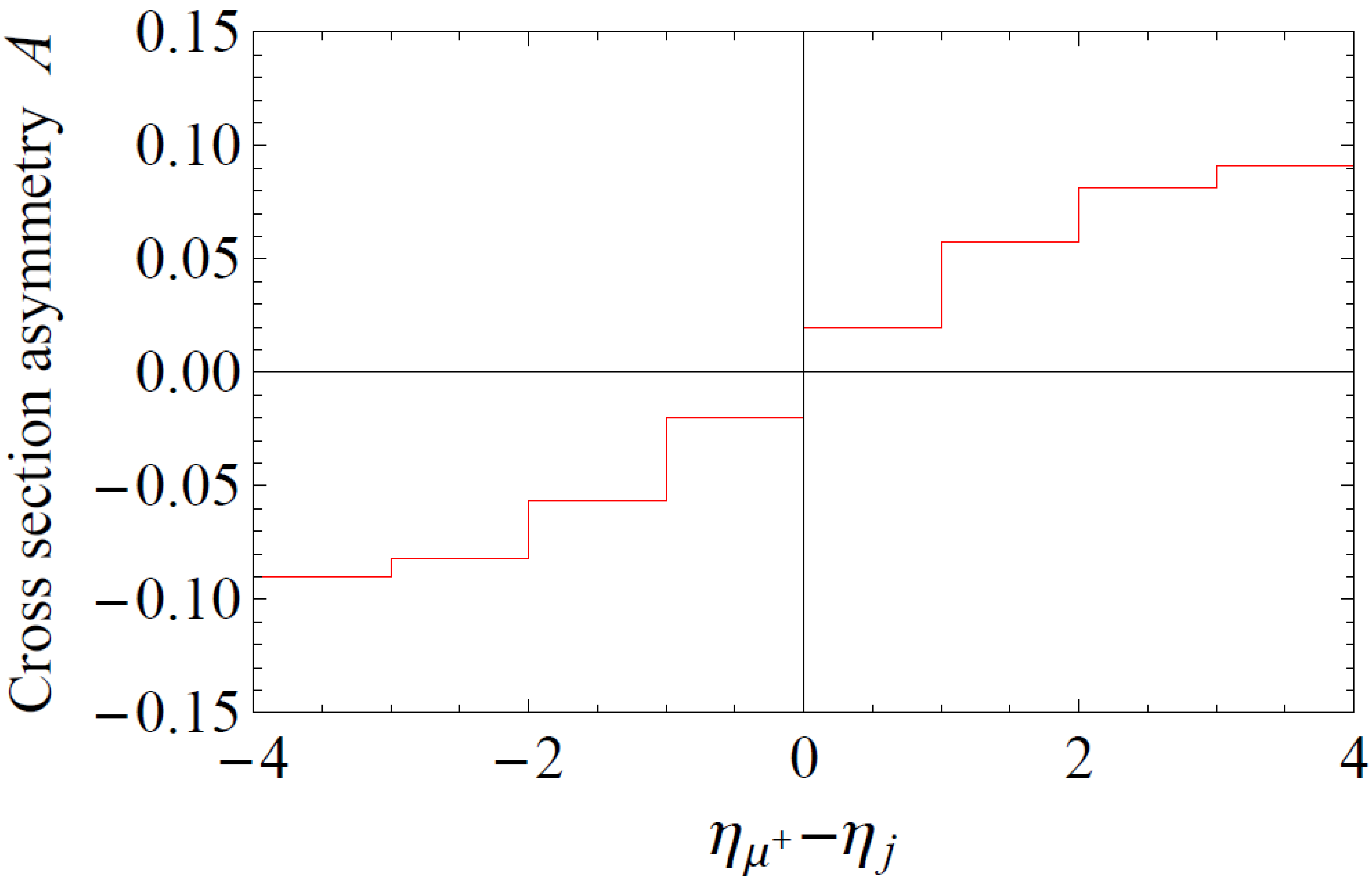
$$\mathrm{sgn}(\,(\vec{q}_l)_y\,) = -\mathrm{sgn}(\,\vec{p}_{\mu T}\times\vec{\not{p}}_T\cdot\vec{n}_z^{\mathrm{lab}}\,)$$

Result of Parton-level Analysis

The cross section asymmetry

$$A \equiv \frac{\sigma(\text{events with } (\vec{q}_l)_y > 0) \; - \; \sigma(\text{events with } (\vec{q}_l)_y < 0)}{\sigma(\text{events with } (\vec{q}_l)_y > 0) \; + \; \sigma(\text{events with } (\vec{q}_l)_y < 0)}$$

in each bin of $\eta_{\mu^+} - \eta_j$ is as below.



Detector-level Analysis

- Since T-odd terms appear at one-loop level, we need a Monte Carlo event generator based on one-loop level calculation of matrix elements, to do a detector-level Monte Carlo simulation.
  Such an event generator is made available recently, which is called "**MadGraph5_aMC@NLO**". J. Alwall *et al.* (2014)

- We use "**MadGraph5_aMC@NLO**" to generate
  $pp \to W^+(\to \mu^+ \nu_\mu) + 1\ \mathrm{jet}$ events with $\sqrt{s} = 8\ \mathrm{TeV}$,
  "**HERWIG6**" to simulate parton showering and hadronization, G. Corcella *et al.* (2010)
  and "**PGS4**" to simulate detector responses and jet clustering. J. Conway *et al.* (2012)
  Jet clustering is done with *Anti-*k_T algorithm with $\Delta R = 0.4$.

- The same cuts as in parton-level analysis are applied to select event.

Scale Uncertainty

- We estimate the uncertainty of theoretical predictions due to the scale choice by varying the renormalization and factorization scales as $q_T/2 \ < \ \mu_R = \mu_F \ < \ 2q_T$.

- Also, we use another Monte Carlo simulator, *"LOMC"*, which calculates the matrix elements at the leading order, for comparison.

"LOMC":

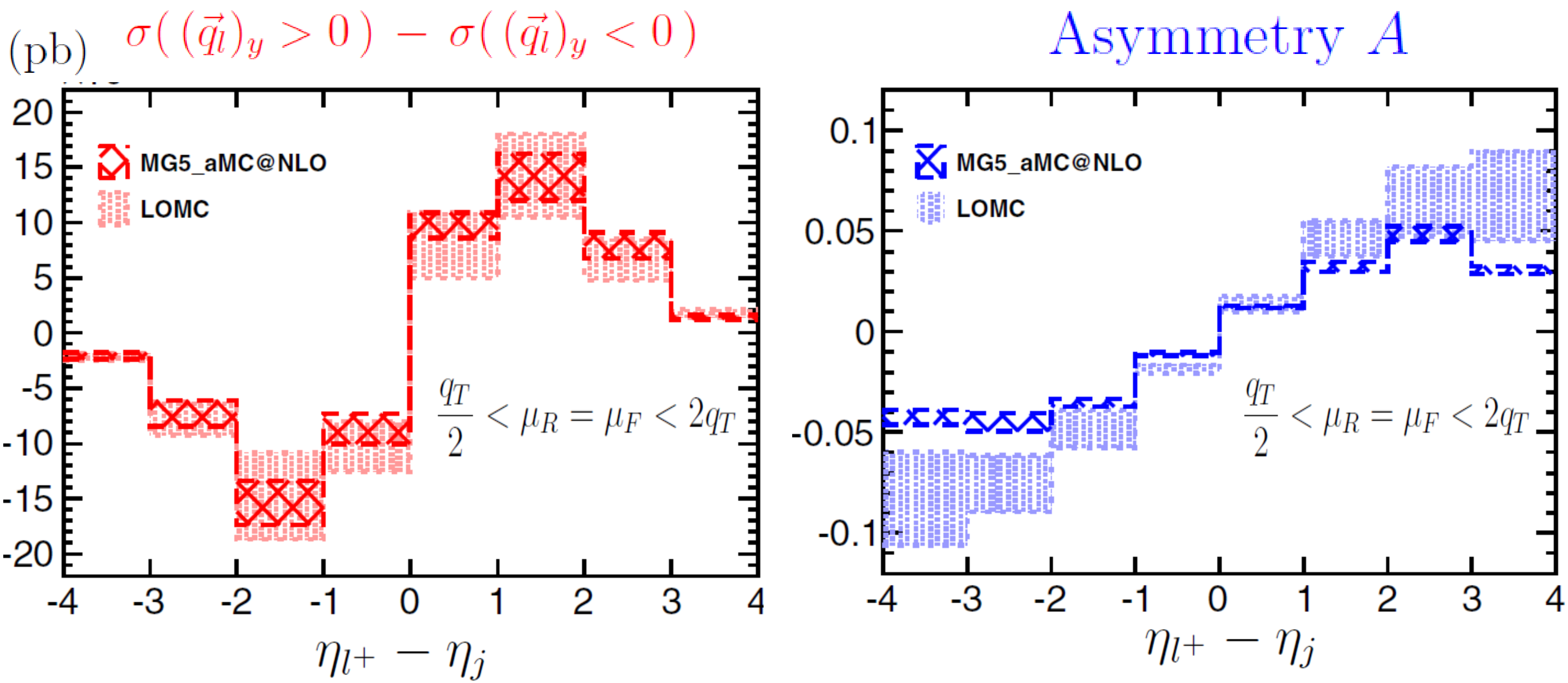
T-even terms (that appear at tree level) are calculated at tree level.

T-odd terms (that appear at one-loop) are calculated at one-loop.

"MadGraph5_aMC@NLO":

Both T-even and T-odd terms are calculated at one-loop.

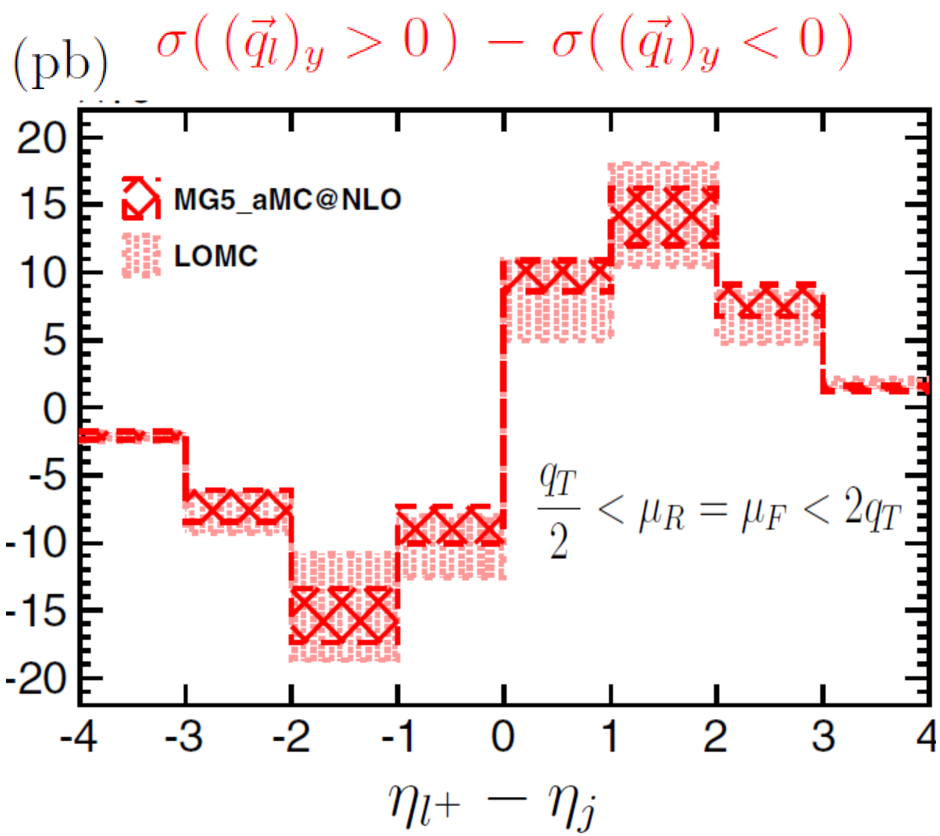
Result of Detector-level Analysis

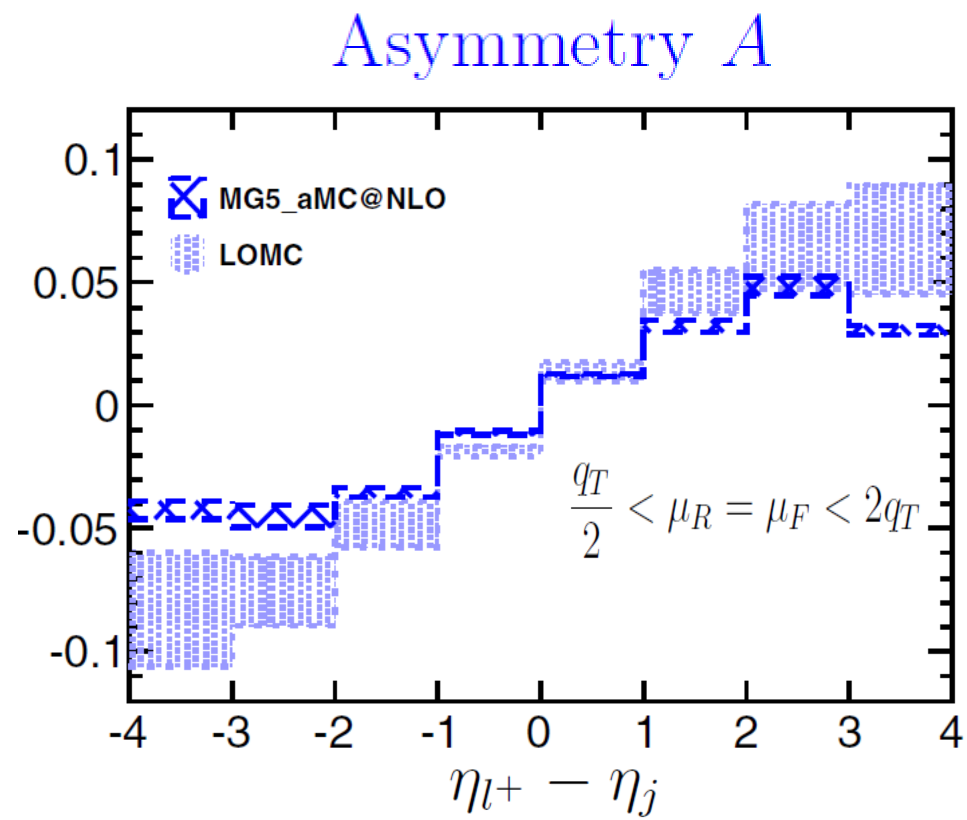


- Asymmetry can be **as large as 5 % even at detector-level**.

- Scale uncertainty is under control with *MadGraph5_aMC@NLO*.
- Background from $pp \to W^+(\tau^+\nu) + \text{jet}$, $\tau^+ \to \mu^+\nu$ events affects the asymmetry by up to 2%.

Result of Detector-level Analysis





With $20\ \mathrm{fb}^{-1}$ of data, the statistical error of the asymmetry A,

$\delta A = \sqrt{(1 - A^2)/N_{\mathrm{evt}}}$, is 0.11 % , 0.15 %, 0.25 %, 0.45%

for $|\eta_{l^+} - \eta_j|$ bins of [0,1], [1,2], [2,3], [3,4].

Summary

- Absorptive part of a scattering amplitude can be measured through T-odd asymmetry of the cross section.
- We have focused on $p\,p \rightarrow W^+ + \mathrm{jet}$ process, where the absorptive part is calculable with perturbative QCD, and study the asymmetry of $\vec{p}_{p1} \times \vec{p}_{W^+} \cdot \vec{s}_\perp$.
- We have done detector-level Monte Carlo simulations of $p\,p \rightarrow W^+(\mu^+, \nu_\mu) + \mathrm{jet}$ process for 8 TeV LHC, and shown that T-odd asymmetry is observable with negligible statistical error with $20\ \mathrm{fb}^{-1}$ of data.